Design of two- and three-element diffractive Keplerian telescopes

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Design procedures for simple two- and three-element diffractive telescopes, suitable for monochromatic applications, are described. We obtained the basic configuration for the two-element design analytically by solving design equations to set the Seidel aberrations to target values. Computer optimization is used to complete the design of the doublet and triplet telescopes. The two- and three-element designs exhibit similar optical performance and diffraction efficiency. We show that diffraction-limited performance can be obtained from these all-diffractive systems.

I. Introduction

Recent advances in fabrication procedures such as diamond machining, photolithography, and laser writers have made the production of high-efficiency surface-relief diffractive lenses practical. These diffractive optical elements are also known as kinoforms, binary lenses, or phase Fresnel lenses. Because these lenses operate by using the principles of diffraction, they are highly dispersive, although they can be used in combination with conventional optical elements to form broadband imaging systems. For monochromatic operation, however, all-diffractive systems can provide an alternative to refractive-reflective systems. The phase functions defining the diffractive lenses can be designed to provide a high level of aberration control from relatively simple systems. Because these diffractive lenses are thin, surface-relief structures, all-diffractive systems can weigh much less than conventional refractive systems. Using diffractive elements can allow for the design of systems with large collecting apertures. Also, these surface-relief diffractive lenses can be easily replicated from a master element.

We consider the design and performance of simple two- and three-element afocal diffractive telescopes to be used with monochromatic illumination as an attachment to a scanning imaging system, such as an active infrared system. We first present the design equations for the correction of the Seidel aberrations of a two-element telescope. The higher-order aberrations are corrected by using lens design optimization techniques. Section II also contains an analysis of an optimized 5X system and a discussion of the pupil aberrations of the telescope. Manufacturing considerations may require the use of a three-element design, as discussed in the next section. We conclude, in Section IV, by comparing the performance of the two- and three-element designs and analyzing the considerations for choosing between them.

II. Design of a Two-Element Keplerian Telescope

The simplest afocal system is two lenses separated by the sum of their focal lengths. Since the telescopes that concern us are not for visual applications, but rather as attachments for scanning imaging systems, we require that the telescope have an external exit pupil. Thus a Keplerian design of two positive elements is needed, as shown in Fig. 1. With two elements we have four degrees of freedom at the third-order design stage, namely, the substrate bending and fourth-order phase coefficient of each lens. (The second-order phase coefficient is determined by the focal length.) We can use these construction parameters to control four Seidel aberrations. Since diffractive lenses have a Petzval contribution of zero, the four degrees of freedom provide complete control of the Seidel aberrations.

We are interested in rotationally symmetric lenses that can be described by a phase function of the form

$$\Phi(r) = 2\pi(Ar^2 + Gr^4 + \cdots),$$

where $r$ is the radial coordinate in the tangent plane.
where \( c_{\text{sub}} \) is the curvature of the lens substrate and \( u \) and \( u' \) are the paraxial angles of the paraxial marginal ray entering and leaving the lens, respectively.

If the stop is not at the lens, the total contribution to the Seidel aberrations is given by use of the stop-shift formulas:

\[
S_n^* = S_n + \frac{y}{y} S_n
\]

\[
S_m^* = S_m + 2\frac{y^2}{y} S_m + \left[ \frac{y^3}{y} \right] S_n
\]

\[
S_p^* = S_p + \frac{y^2}{y} (S_m + S_p) + 3 \left[ \frac{y^3}{y} \right] S_n + \left[ \frac{y^4}{y} \right] S_p
\]

Where \( y \) is the highest height of the paraxial chief ray at the lens.

With these equations we can complete a third-order design of a two-element telescope. The stop is in contact with the objective lens, so the stop-shift equations need only be applied to the stop-in-contact aberration coefficient contributions from the eyepiece lens. At the eyepiece a paraxial ray trace shows that the stop-shift parameter is

\[
\frac{y}{y} = \frac{y(M - 1)}{y_{\text{obj}}},
\]

where \( M \) is the paraxial afocal magnification given by \( M = y_{\text{obj}}/y_{\text{eye}} = -f_{\text{obj}}/f_{\text{eye}} = -\phi_{\text{eye}}/\phi_{\text{obj}} \) the paraxial marginal ray height at the objective, \( y_{\text{eye}} \) is the paraxial marginal ray height at the eyepiece, \( \phi_{\text{obj}} \) is the power of the objective lens, \( \phi_{\text{eye}} \) is the power of the eyepiece lens, \( f = 1/\phi \) is the focal length of each lens, and \( u \) is the paraxial field angle in object space. Since the Petzval sum is zero for an all-diffractive system, we have only four third-order aberrations to control. We also have four degrees of freedom left in the design: the bending and the fourth-order phase coefficient of each lens. We can solve a system of four equations in these four unknowns to provide the third-order design. As target values for the Seidel aberrations we choose zero for the spherical aberration, coma, and astigmatism and sufficient distortion to provide a constant real-ray angular magnification. This distortion is needed for coupling to a constant-angular-velocity scanning system that will provide the final real image from the telescope. We find this necessary amount of distortion by considering the angular aberration that is necessary for a real-ray angular magnification of \( \theta' / \theta \). We denote the paraxial chief ray angle in object space by \( \theta \) and in image space by \( \theta' \). Then the real-ray angles as a function of the fractional object height \( h \) are \( \theta = \tan^{-1}(\theta h) \) and \( \theta' = \tan^{-1}(\theta' h) \). The necessary angular aberration \( \alpha \) is the difference between the desired
relationship $\theta' = M \theta$ and the paraxial relationship $\tan(\theta') = M \tan(\theta) = M \bar{u}h$ or equivalently $\theta' = \tan^{-1}(M \bar{u}h)$. Thus

$$\alpha = M \theta - \tan^{-1}(M \bar{u}h) = M \tan^{-1}(\bar{u}h) - \tan^{-1}(M \bar{u}h).$$

We can expand the $\tan^{-1}$ functions in Eq. (8) to find the necessary amount of third-order distortion. Using the relation $\tan^{-1}(z) = z - z^3/3 + z^5/5 - \ldots$, we find that

$$\alpha = M \left( \bar{u}h - \frac{M^3 \bar{u}h}{3} + \ldots \right) - \left( M \bar{u}h - \frac{M^3 \bar{u}h}{3} + \ldots \right)$$

$$= \frac{1}{3} M(M^2 - 1) \bar{u}^2 h^3 + \ldots.$$  (9)

The Seidel sum $S_v$ is related to the angular aberration $\alpha$ by $S_v = (2 H/\bar{u}^2)\alpha$. Using $H = -\bar{u}$, $\gamma_{obj} \bar{u}' = M \bar{u}$, and Eq. (9), we find the necessary third-order distortion to be

$$S_v = \frac{2}{3} \gamma_{obj} \bar{u}^2 (1 - M^2).$$  (10)

Solving the set of four equations,

$$S_i = S_ii = 0, \quad S_v = \frac{2}{3} \gamma_{obj} \bar{u}^2 (1 - M^2),$$

for the four unknowns $B_{obj}$, $B_{eye}$, $G_{obj}$, and $G_{eye}$, we find that the solutions are

$$B_{obj} = \frac{-2(4 + M)}{3(1 - M)};$$  (12a)

$$B_{eye} = \frac{-2(4 + M)}{3(1 - M)};$$  (12b)

$$G_{obj} = \frac{\phi_{obj}(M^2 - 7M + 1)}{72 \lambda (1 - M)^2};$$  (12c)

$$G_{eye} = \frac{\phi_{eye}(M^2 - 7M + 1)}{72 \lambda (1 - M)^2}.$$  (12d)

At this stage of the design the performance is limited by higher-order aberrations, usually fifth-order spherical aberration and astigmatism for systems of practical speed and field angle.

As an example of the performance that can be expected from these simple two-element telescopes at the third-order design stage, we designed a telescope with a magnification $M = -5$ for use at $\lambda_e = 10.6 \mu m$ with an entrance pupil diameter of 100 mm. The system specifications, resulting from the design procedure given here, are given as follows:

- $f_{obj} = 200 \text{ mm}$, $R_{obj} = 189.5 \text{ mm}$,
- $A_{obj} = -0.2358 \text{ mm}^{-2}$, $G_{obj} = 2.775 \times 10^{-4} \text{ mm}^{-4}$,
- clear aperture $a_{obj} = 100 \text{ mm}$, $f/#_{obj} = 2.0$,
- $f_e = 40 \text{ mm}$, $R_{eye} = 720 \text{ mm}$,
- $A_{eye} = -1.1792 \text{ mm}^{-2}$, $G_{eye} = 3.469 \times 10^{-4} \text{ mm}^{-4}$,
- clear aperture $a_{eye} = 53.6 \text{ mm}$, $f/#_{eye} = 0.75$.

In the above specifications $R_{eye}$ is the radius of curvature of the lens substrate and $f/#$ is the construction $f$-number of the lens, which is defined as the focal length divided by the clear aperture. Figure 2 shows the wave-front aberration for the telescope that is evaluated on-axis and at an object space half-field angle of 4 deg. These curves show the general trend that the major uncorrected aberrations are fifth-order spherical aberration and astigmatism.

The fifth-order aberrations can be corrected by adding a sixth-order phase term ($H\pi$) to each lens and optimizing these coefficients by using an optical design software program. We used the program SUPEROSLO and optimized the two sixth-order phase function terms by using a simple merit function consisting of the squared sum of the fifth-order aberration coefficients. The resulting sixth-order phase coefficients are $H_{obj} = 7.368 \times 10^{-15} \text{ mm}^{-6}$ and $H_{eye} = -2.161 \times 10^{-14} \text{ mm}^{-6}$. Figure 3 shows the rms wave-front aberration for field angles of up to 4 deg in object space. As is evident from the figure this simple telescope provides diffraction-limited performance across the field. Figure 4 shows a plot of the image space field angle $\theta'$ as a function of the object space field angle $\theta$. This linear relationship is due to distortion, as discussed above.

Since this telescope is designed to be coupled to a scanning imaging system, the telescope needs to have acceptable pupil imagery. For this purpose an analysis of the third-order pupil aberrations is instructive. For diffractive lenses the Seidel pupil aberration coefficients are most easily derived first by evaluating the stop-in-contact contribution and then by applying the stop-shift equations for the pupil aberrations. The stop-in-contact forms may be found by taking the limit as the refractive index goes to infinity for a refractive thin lens. This technique has proved effec-
CII = 1I1 + (BS + Sil) + ( )'Si,

SV* = Sv + S.

Field Angle (degrees)

Fig. 3. Root-mean-square wave-front error for the optimized two-element telescope with an additional sixth-order phase term.

ative in evaluating the image aberrations. The pupil Petzval curvature is equal to the image Petzval curvature and is thus equal to zero for all-diffractive systems. We denote the pupil aberrations by a barred symbol. With the aperture stop in contact the pupil aberrations are as follows:

Pupil spherical aberration,

$$\bar{S}_t = 0;$$

(13a)

Pupil coma,

$$\bar{S}_n = 0;$$

(13b)

Pupil astigmatism,

$$\bar{S}_m = 0;$$

(13c)

Pupil distortion,

$$\bar{S}_v = -\frac{1}{2} H y^5 \Phi^5 B.$$ 

(13d)

The stop-shift equations for the pupil aberrations are

$$\bar{S}_t^* = \bar{S}_t + \frac{y}{y} (3\bar{S}_n + \bar{S}_v) + \left(\frac{y^2}{y} (3\bar{S}_m + 2\bar{S}_v + 3\bar{S}_m) + \left(\frac{y^3}{y} (3\bar{S}_n + 2\bar{S}_v + 3\bar{S}_m) + \left(\frac{y^4}{y} (3\bar{S}_m + 2\bar{S}_v + 3\bar{S}_m) + \left(\frac{y^5}{y} (3\bar{S}_n + 2\bar{S}_v + 3\bar{S}_m) + \bar{S}_m \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) 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where \( \lambda_0 \) is the design wavelength and \( f/\# \) is the construction \( f \)-number of the lens. For the example telescope this means that the finest zone has a width of \( \sim 16 \mu m \). This zone width is, however, manufacturable by using current manufacturing technologies.\(^{1-5}\)

By splitting the eyepiece into two separated elements, as depicted schematically in Fig. 5, we can reduce the speed of each eyepiece lens and hence increase the minimum zone width. However, the speed of each element in the two-lens eyepiece can be reduced only by a factor of \( \sim 2 \) from the speed of the equivalent single-element eyepiece. This conclusion is illustrated in Fig. 6, which shows the construction \( f \)-number (focal length/clear aperture) of the two lenses in the eyepiece relative to the \( f \)-number of the single eyepiece lens (in this case \( f/0.75 \)) as a function of the back focal distance of the eyepiece. For this exercise we assumed that the focal lengths of the two eyepiece lenses are equal. The figure illustrates the general conclusion that only a factor of \( \sim 2 \) in the \( f \)-number (and hence in the minimum zone width) can be obtained by splitting the eyepiece lens.

We have found that a three-element diffractive telescope with sixth-order phase functions can be optimized for diffraction-limited performance, just as for the two-element designs. For most three-element systems a simple merit function consisting of the sum of the squared third-and fifth-order image aberration coefficients is sufficient for a lens design program optimization. The variables for the optimization are the bendings and the fourth- and sixth-order phase coefficients of the three lenses. The rms wave-front error as a function of the field angle for a three-element telescope is shown in Fig. 7. This telescope has the same first-order properties as the two-element telescope that is described in the previous section. Each lens in the eyepiece has a focal length of 70 mm and a construction \( f \)-number of approximately \( f/1.5 \).

**IV. Comparison of Two- and Three-Element Designs**

As we see from the previous sections, little is to be gained in terms of increased optical performance by using a three-element design rather than a two-element telescope. There may be configurations of a large field angle or faster objectives where the additional element is necessary to achieve the desired performance. In situations like the one discussed in this paper, where the two- and three-element performances are essentially the same, the question is whether one configuration is preferred over the other for reasons other than optical performance. One consideration that we mentioned earlier is the manufacturability constraints that are imposed by the small feature sizes of zones near the edge of the lenses. This will probably not be much of a problem in the 8–12-\( \mu m \) region but may be important if shorter wavelengths and smaller diffracting zones are to be used.

Another consideration is the overall diffraction efficiency of the systems. For these reasonably fast diffractive lenses, we expect the diffraction efficiency to depart significantly from the scalar prediction of unity as a result of the large wavelength-to-grating-period ratios. For example, by using the DIFFRACT\(^2\) program from MIT Lincoln Laboratory, which calculates grating efficiencies by using rigorous electromagnetic coupled wave theory, we found an empirical formula for the local diffraction efficiency for diffractive lenses that are formed on a germanium substrate with a \( \lambda/4 \) antireflection coating. We obtained this formula by using DIFFRACT to calculate the first-order diffraction efficiency of a germanium substrate transmission grating for a number of grating periods corresponding to radial zones of the diffractive lens. The relationship between grating frequency and radial position is found by rewriting approximation (17). At any radial point \( r \) on the lens, the \( f \)-number at that point is \( f/\# = f/(2r) \), where \( f \) is the focal length. By denoting the zone width at \( r \) by \( s \), approximation (17) can be rewritten as

\[
\frac{r}{f} = \frac{\lambda_0}{s}.
\]  

(18)

**Fig. 5.** Layout of the three-element diffractive telescope.

**Fig. 6.** The \( f \)-numbers of the two eyepiece lenses, normalized to the \( f \)-number of the singlet eyepiece lens, as a function of the back focal distance of the doublet eyepiece.

**Fig. 7.** Root-mean-square wave-front error for the optimized three-element telescope.
Fitting a line to the results of the DIFFRACT calculation for first-order efficiency as a function of $\lambda_0/s$, i.e., as a function of $r/f$, yields the following empirical formula:

$$\eta_{\text{line}}(r) = 1 - 0.32 \left( \frac{r}{f} \right).$$

(19)

In approximation (19) $r$ is the radial coordinate on the lens and $f$ is the focal length of the lens. We calculated approximation (19) by assuming a classical blazed grating groove shape with a blaze height of $\lambda_0/(n - 1)$, where $\lambda_0$ is the design wavelength (10.6 $\mu$m in this case) and $n$ is the refractive index of the substrate. Integrating approximation (19) over the illuminated portion of each lens for each field position gives the integrated total efficiency. For the examples in this paper, the integrated efficiencies are as follows:

- **on-axis**
  - $\eta_{\text{doublet}} = 0.896$, $\eta_{\text{triplet}} = 0.897$

- **4 degrees off-axis**
  - $\eta_{\text{doublet}} = 0.814$, $\eta_{\text{triplet}} = 0.818$

Thus we see the interesting result that the effects of the reduced efficiency of the faster eyepiece lens in the two-element telescope and of the increased number of lenses in the three-element design are approximately the same.

**V. Conclusion**

For monochromatic applications simple two- and three-element diffractive Keplerian telescopes can provide a diffraction-limited performance. The feasibility of the two-element design depends on field angle, lens speed, fabrication technique, and overall diffraction efficiency. In many cases the two-element design exhibits a performance and diffraction efficiency that are similar to those of the three-element design. If it can be manufactured, the two-element design can provide high performance with a simple layout, less weight, and fewer lenses to align.

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**References and Notes**


18. Ref. 12, pp. 147–148.

19. SUPER-OSLO is a product of Sinclair Optics, Inc., 6780 Palmyra Road, Fairport, N.Y. 14450.

