Useful Formulas for Diffractive Lens Surfaces

Diffracted Wavefront: $$OPD(r) = \frac{m\lambda}{p\lambda_0}\left(c_1 r^2 + c_2 r^4 + c_3 r^6 + \cdots\right) \quad \textit{OPD equation}$$

Blazed Zone Depth: $$d_0 = \frac{p\lambda_0}{\left|n_t(\lambda_0) - n_i(\lambda_0)\right|} \quad \textit{design depth}$$

Diffraction Efficiency: $$\eta = \mathrm{sinc}^2(\alpha - m) = \left\{\frac{\sin[\pi(\alpha - m)]}{\pi(\alpha - m)}\right\}^2 \quad \textit{scalar theory}$$

where, $$\alpha(\lambda) = \frac{p\lambda_0}{\lambda}\left|\frac{n_t(\lambda) - n_i(\lambda)}{n_t(\lambda_0) - n_i(\lambda_0)}\right| \quad \textit{for small angles, at design depth}$$

or, $$\alpha(\lambda, d) = \frac{d\left|n_t(\lambda) - n_i(\lambda)\right|}{\lambda} \quad \textit{for small angles, at any depth}$$

or, $$\alpha(\lambda, d, \theta) = \frac{d\left|n_t(\lambda)\cos\theta_t - n_i(\lambda)\cos\theta_i\right|}{\lambda} \quad \textit{general case}$$

where: $$n_t(\lambda)\sin\theta_t = n_i(\lambda)\sin\theta_i + \frac{m\lambda}{\Lambda} \quad \textit{the grating equation}$$

Key to symbols:

$OPD(r)$ = Optical path difference in same units as r [divide by λ for waves]

r = Radial position of interest $r = \sqrt{x^2 + y^2}$

m = Operating diffraction order

λ = Operating wavelength

p = Design order (diffracted order for which zone depth is optimized)

λ_0 = Design wavelength for diffractive surface (for which depth is usually optimized)

c_1, c_2, … = Coefficients of OPD (phase) polynomial for diffracted wavefront

Λ = Local diffraction zone width (grating period) Λ_{min} = Minimum zone width

d = Diffraction zone peak-to-valley depth

d_0 = Zone depth for maximum efficiency for the design wavelength at normal incidence

$n_i(\lambda)$, $n_t(\lambda)$ = Refractive index on the incident (i) and diffracted (t) side of the grating

η = Scalar diffraction efficiency

α = Detuning factor for depth errors, wavelength change and material dispersion

θ_i, θ_t = Angles of incidence (i) and diffraction (t), relative to local surface normal

Paraxial Quantities:

$$f(\lambda) = \frac{-p\lambda_0}{2m\lambda c_1} \quad \text{and} \quad \Lambda_{\min} \approx 2\lambda_0(f/\#) \quad \textit{ignoring higher order terms}$$

Hybrid (refractive/diffractive) achromatic singlet lens:

	Refractive Component	Diffractive Component
Component power	$\Phi_{ref} = \frac{\nu_{ref}}{\nu_{ref} - \nu_{dif}} \Phi_{total}$	$\Phi_{dif} = \frac{\nu_{dif}}{\nu_{dif} - \nu_{ref}} \Phi_{total}$
Abbe ν-number	$\nu_{ref} = \frac{n(\lambda_{mid}) - 1}{n(\lambda_{short}) - n(\lambda_{long})}$	$\nu_{dif} = \frac{\lambda_{mid}}{\lambda_{short} - \lambda_{long}}$
Partial dispersion	$P_{ref} = \frac{n(\lambda_{short}) - n(\lambda_{mid})}{n(\lambda_{short}) - n(\lambda_{long})}$	$P_{dif} = \frac{\lambda_{short} - \lambda_{mid}}{\lambda_{short} - \lambda_{long}}$

Key to symbols:

m = Diffracted order of interest

λ = Wavelength of interest

p = Design order (diffracted order for which zone depth is optimized)

λ_0 = Design wavelength for diffractive surface

c_1, c_2, ... = Coefficients of OPD (phase) polynomial for diffracted wavefront

f = Focal length (paraxial)

Λ = Local diffraction zone width (grating period)

$f/\#$ = F-number = f / (lens diameter)

Φ_{total} = Total hybrid lens power (reciprocal of focal length) at design wavelength

Φ_{ref} , Φ_{dif} = Individual component powers at design wavelength

ν_{ref} , ν_{dif} = Abbe ν-numbers for refractive and diffractive components

P_{ref} , P_{dif} = Partial dispersion values for refractive and diffractive components

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