Diffractive–refractive behavior of kinoform lenses

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The connection between the diffractive behavior and the refractive behavior of kinoform lenses is investigated. The image-forming capabilities of the diffractive element are found to be expressed as an interference pattern that is due to a set of associated refractive lenses corresponding to each zone of the kinoform. The coefficient modulating the contribution of each refracting zone is determined. The term linking the point-spread function of a refractive lens and a diffractive lens is also obtained. Spectral and spatial aspects are compared as the diffractive element approaches the refractive limit. © 1997 Optical Society of America

1. Introduction

Holographic elements are well known for their capacity to realize complex operations not possible to carry out with purely refractive elements. The ability to manipulate the local amplitude and phase of an incoming wave has stimulated the use of holographic elements in applications such as beam shaping and optical pattern recognition, among others. A class of surface-relief diffractive elements known as kinoforms has been developed that behave as refractive lenses under certain circumstances.[1] A fundamental difference between these two lenses resides in what defines the image-forming capabilities of each element. On one hand, a refractive lens (as implied by its name) relies on the refraction effects that take place when the incident light interacts with its surface. In this case, light can be viewed as geometrical rays obeying simple trigonometric relations. A diffractive element, on the other hand, forms an image by a coherent superposition of the light that comes from each of its constituent zones. Although refraction also takes place, it is an interference effect that determines the formation of each image. In theory, a diffractive lens should perform exactly as a refractive one at a specific wavelength, thereby allowing the design of optical systems of reduced weight and more compactness. Furthermore, because fabrication techniques are under constant improvement, mass-production costs are also declining. A large amount of chromatic aberrations, however, has restrained the extensive use of purely diffractive lenses in broadband applications. In such instances, hybrid refractive–diffractive elements[2] can be successfully integrated to compensate for chromatic aberrations, with the diffractive element being used instead of expensive flint glasses.

Although a diffractive lens fundamentally operates through interference, it also combines refraction effects to some degree. The refractive limit is generally associated with the applicability of the laws of geometrical optics in the sense that the dynamics of image formation can be completely understood from a ray picture. When one considers a diffractive lens, it is apparent that refraction can be used to explain the lens behavior when only one zone is present, strictly speaking. For a greater number of zones, interference becomes the primary factor in image formation. The question then becomes how the diffractive nature of the lens relates to the refractive effects that occur in each zone, or how refractive properties combine to yield the behavior of the diffractive lens. There has been some recent interest[3,4] in the transition from diffractive to refractive behavior in micro-optical diffractive elements, largely motivated by the introduction of a new class of lenses known as multiorder[5] or harmonic[6] lenses. In this study we examine the behavior of diffractive lenses, with respect to both refraction and interference effects, in an effort to clarify how these two phenomena interrelate. Previous studies have focused on gratings[3] or the associated grating picture of a diffractive lens together with numerical simulations.[4] We adopt a different approach that permits the treatment of diffractive lenses as finite objects and at the same time yields analytical results. In what follows, we investigate the diffractive–refractive behavior of the diffractive

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lens in terms of the on-axis scalar field. We find that the diffraction pattern of the holographic lens can be seen as the interference of a number of associated refractive lenses modulated by a complex coefficient. In a specific case, the term linking the diffractive and the refractive element point-spread function can be determined explicitly.

2. Diffractive–Refractive Behavior

In this paper we focus on the paraxial design of a diffractive lens,[1] i.e., the parabolic blaze profile, because in this case it is possible to carry out useful analytical calculations. For more complicated profiles it is not generally possible to obtain closed-form solutions for the diffraction integral, and one has to resort to numerical simulations. Thus, to gain a better understanding of our problem we concentrate on the parabolic lens design, which in addition to its practical importance also allows us to demonstrate clearly its diffractive–refractive behavior.

The phase function of a diffractive lens within a paraxial design is written as

$$\phi(r) = 2\pi\alpha\left(n - \frac{r^2}{2\lambda_0 f}\right), \qquad r_{n-1} \leq r < r_n, \quad (1a)$$

$$r_n{}^2 = 2\lambda_0 f n, \qquad n = 1, 2, 3, \ldots, N, \quad (1b)$$

where r_n localizes the boundaries of each zone, $r_0 = 0$ by definition, λ_0 is the design wavelength, f is the focal length, N is the total number of zones, and parameter α is given by

$$\alpha = \left(\frac{\lambda_0}{\lambda}\right)\left[\frac{n(\lambda) - 1}{n(\lambda_0) - 1}\right], \quad (2)$$

where $n(\lambda)$ is the index of refraction of the lens material for a given wavelength λ.

The diffractive lens defined by Eq. (1) has a very small thickness equal to $\lambda_0/[n(\lambda_0) - 1]$, corresponding to only a few wavelengths. This particular design cannot be used in broadband applications because the focal dependence on wavelength as expressed by

$$f_m = \frac{\lambda_0 f}{\lambda m}, \quad (3)$$

with m as an integer, causes the appearance of additional diffracted orders for $\lambda \neq \lambda_0$. Therefore, there is a reduction in efficiency for the main diffracted order (usually the first), giving rise to a degradation of the image quality. To address this problem, researchers have proposed a lens design based on the use of higher diffracted orders.[5,6] The new scheme involves the introduction of a parameter p, which essentially determines a set of diffracted orders associated with different wavelengths that come to a single focus. Consequently, singlets can be achromatized for those certain values of incident wavelength, although one cannot guarantee high performance with even slight deviations from the design conditions. A higher-order diffractive lens can be defined by means of the following phase function:

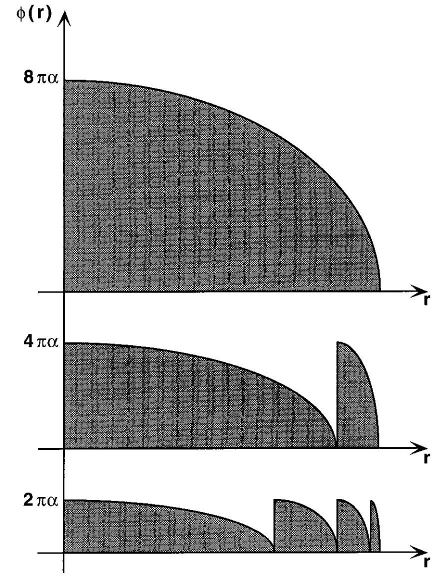

Fig. 1. Iterative process showing how the limits of a diffractive kinoform and a purely refractive lens are obtained. Only a cross section of phase function $\phi(r)$ is illustrated. In each step the semiaperture is kept constant while maximum phase transmission $2\pi\alpha p$ increases, where p is an integer. When the number of zones $N = 1$, we have a purely refractive lens. As N increases, with a subsequent decrease in p, the diffractive properties of the element become predominant.

$$\phi(r) = 2\pi\alpha p\left(n - \frac{r^2}{2\lambda_0 f p}\right), \qquad r_{n-1} \leq r < r_n, \quad (4a)$$

$$r_n{}^2 = 2\lambda_0 f p n, \qquad n = 1, 2, 3, \ldots. \quad (4b)$$

In addition to permitting diffractive singlet achromatization, a higher-order element also presents larger zones, placing less stringent demands on fabrication. It has been recognized[3,4] that the transition region from diffractive to refractive optics can be readily understood through the concept of a higher-order element, that is, a variable phase-height element. The idea is very simple and applies not only to lenses but also to gratings. Consider a diffractive lens with constant design parameters: fix the clear aperture, and increase parameter p in Eqs. (4). The number of zones will then decrease as illustrated in the sequence of Fig. 1. The refractive aspects of the element become more and more important as p increases, constituting what we would call a totally refractive element when only one zone remains, provided the zone size is considerably larger than the incident wavelength, as it is in many practical cases.

The higher-order lens suggests a general formulation of a diffractive lens phase function that can be written as follows:

$$\phi(r) = 2\pi\alpha\left(g_n - \frac{r^2}{2\lambda_0 f}\right), \qquad r_{n-1} \leq r < r_n, \quad (5a)$$

$$r_n{}^2 = 2\lambda_0 f g_n, \qquad n = 1, 2, 3, \ldots, \quad (5b)$$

where g_n is a function of an integer argument. The main reason for the generalization in Eqs. (5) will become clear in the following developments when the refractive role of each zone is made evident. In particular, we have the conventional, $g_n = n$, and the higher-order, $g_n = pn$, cases for the usual diffractive lens designs.

To elicit the diffractive–refractive relation, we develop some on-axis calculations based on the Fresnel approximation to the diffraction integral[7] given by

$$\psi(z) = \frac{2\pi \exp(ikz)}{i\lambda z} \int_0^{r_N} \exp[i\phi(r)]\exp\left(\frac{i\pi}{\lambda z} r^2\right) r\mathrm{d}r, \quad (6)$$

where we have made use of the circular symmetry of the elements under consideration and where r_N is the boundary of the outermost zone or the semiaperture of the lens. Use of the phase function of Eqs. (5) in the diffraction integral yields the following, somewhat formidable, expression for the on-axis scalar field:

$$\begin{aligned}\psi_D(z) = 2\pi\left(\frac{\lambda_0 f}{\lambda z}\right) \sum_{n=1}^{N} \exp\left\{i\pi\left[2\alpha g_n + \left(\frac{\lambda_0 f}{\lambda z} - \alpha\right)\right.\right.\\ \left.\left.\times (g_n + g_{n-1})\right]\right\}(g_n - g_{n-1})\mathrm{sinc}\left[\left(\frac{\lambda_0 f}{\lambda z} - \alpha\right)\right.\\ \left.\times (g_n - g_{n-1})\right], \quad (7)\end{aligned}$$

where subscript D indicates a diffractive lens and irrelevant phase terms were ignored.

By using Eq. (6) we can also determine the scalar field of a purely refractive lens with maximum phase height $2\pi\alpha\phi_0$, given by

$$\begin{aligned}\psi_R(z) = 2\pi\left(\frac{\lambda_0 f}{\lambda z}\right)\exp\left\{i\pi\left[2\alpha\phi_0 + \left(\frac{\lambda_0 f}{\lambda z} - \alpha\right)\phi_0\right]\right\}\\ \times \mathrm{sinc}\left[\left(\frac{\lambda_0 f}{\lambda_z} - \alpha\right)\phi_0\right], \quad (8)\end{aligned}$$

with index R indicating a refractive lens. We note that Eqs. (7) and (8) can be combined because the summation over zones in the diffractive scalar field can be seen as a sum over the field caused by local refractive lenses. The result is

$$\psi_D(z) = \sum_{n=1}^{N} \exp\left[i2\pi\left(\frac{\lambda_0 f}{\lambda_z}\right) g_{n-1}\right](g_n - g_{n-1})\psi_R{}^n(z), \quad (9)$$

where $\psi_R{}^n$ is the phase function of a refractive lens of maximum phase height such that $\phi_0 = g_n - g_{n-1}$. Because term $g_n - g_{n-1}$ is directly related to each single zone, the diffractive element can be seen as the interference of N refractive lenses whose characteristics are determined locally by each zone. Notice that this case is different from a lens array, which is more akin to a grating in which each period may be constituted by a refractive lens.

The on-axis field that is due to each associated refractive lens is modulated by the complex interference term,

$$\psi_I{}^n(z) = \exp\left[i2\pi\left(\frac{\lambda_0 f}{\lambda z}\right) g_{n-1}\right](g_n - g_{n-1}), \quad (10)$$

which couples the refractive field amplitude of each zone. The expression of the scalar field as in Eq. (9) explicitly gives the contribution of each zone to form the final interference pattern of the generalized diffractive lens. In the case of a higher-order lens we can see that

$$g_n - g_{n-1} = pn - p(n-1) = p, \quad (11)$$

implying that the refractive field associated with each zone is independent of n and can be taken out of the summation. If we write $\psi_R{}^n(z) = \psi_R{}^p(z)$ to refer to the higher-order lens, we can write Eq. (9) as

$$\begin{aligned}\psi_D(z) &= \psi_R{}^p(z) \sum_{n=1}^{N} p \exp\left[i2\pi\left(\frac{\lambda_0 f}{\lambda z}\right) p(n-1)\right]\\ &= \psi_R{}^p(z)\gamma(z), \quad (12)\end{aligned}$$

where $\gamma(z)$ represents the summation of phase terms over all zones and can be calculated exactly to be

$$\gamma(z) = Np \exp\left[i\pi\left(\frac{\lambda_0 f}{\lambda z}\right)(N+1)p\right]\frac{\mathrm{sinc}[(\lambda_0 f/\lambda z)Np]}{\mathrm{sinc}[(\lambda_0 f/\lambda z)p]}. \quad (13)$$

It is also interesting to investigate how the point-spread function (PSF) of the diffractive element relates to that of the refractive lens. For this purpose, we consider a higher-order lens with maximum phase height $2\pi\alpha p$ and N zones. Infinite conjugate imaging is assumed. The comparison to be carried out here is with a refractive lens of maximum phase height $2\pi\alpha Np$ and the same clear aperture as the kinoform. After straightforward albeit somewhat tedious calculations, which again are based on Eq. (6), we find that the PSF of the diffractive lens, $I_D(z)$, is written in terms of the PSF, $I_R(z)$, of the refractive lens as

$$I_D(z) = I_T(z)I_R(z), \quad (14)$$

where term $I_T(z)$ relating the diffractive–refractive limits can be written as

$$I_T(z) = \prod_{i=1}^{\beta} \frac{\cos^2[2^{i-1}\pi(\lambda_0 f/\lambda z)p]}{\cos^2\{2^{i-1}\pi[(\lambda_0 f/\lambda z) - \alpha]p\}}, \quad (15)$$

assuming that $N = 2^\beta$ for an integer β and $\Pi_{i=1}^{\beta} \alpha_i = \alpha_1\alpha_2 \cdots \alpha_\beta$. Notice that in Eq. (14), term $I_R(z)$ corresponds to the purely refractive limit illustrated in Fig. 1. At $\lambda = \lambda_0$ the transition coefficient Eq. (15) reduces to unity and the diffractive element behaves essentially as a refractive lens. This is expected because both elements are identical at λ_0, except for a modulo $2\pi p$ transformation rule. However, for a wavelength detuning from the design, the transition term becomes important and gives rise to those phenomena that afflict a diffractive element such as re-

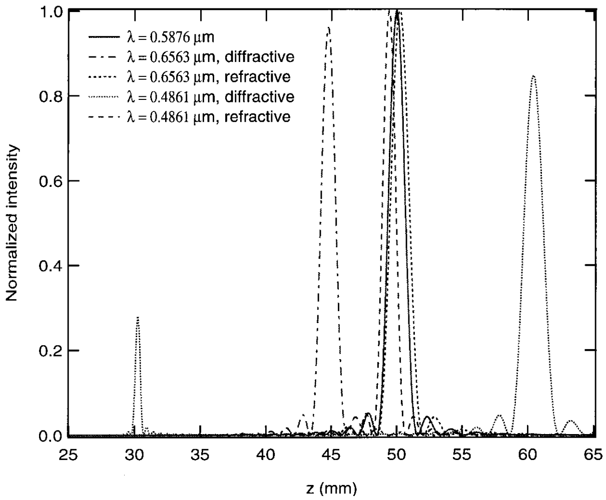

Fig. 2. PSF's illustrating the dispersive nature of a diffractive lens in comparison to a refractive one. The on-axis intensity is shown for three wavelength values: $\lambda = 0.5876$ μm (index of refraction $n_D = 1.5168$ for BK-7 glass), $\lambda = 0.4861$ μm ($n_F = 1.5143$), and $\lambda = 0.6563$ μm ($n_C = 1.5228$). The diffractive lens was designed to operate at $\lambda_0 = 0.5876$ μm with a focal length of 50 mm. In this case $p = 1$ and $N = 32$. The solid curve shows the behavior at the design wavelength, which is identical for both diffractive and refractive elements. Note the opposite sign of the dispersion and the appearance of multiple orders.

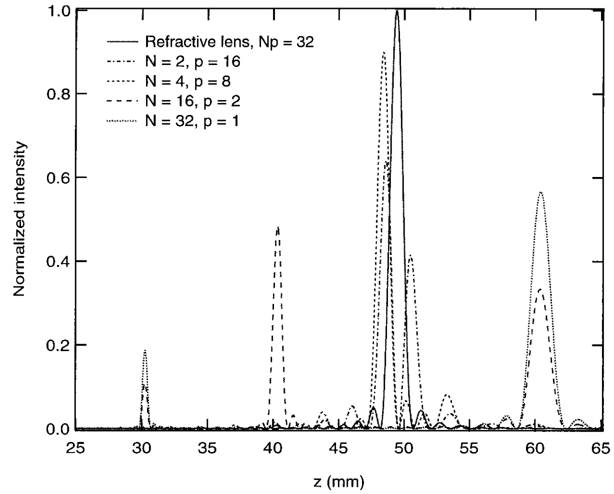

Fig. 3. Evolution of the PSF of a diffractive lens of 32 zones and $p = 1$ toward the refractive limit (or $N = 1$). Design parameters: $\lambda_0 = 0.5876$ μm, $F = 50$ mm, and $\lambda = 0.4861$ μm. For a meaningful comparison, product Np is set to a constant equal to 32. Note that as N decreases (p increases) the diffractive pattern tends to approximate the spectral behavior of the refractive lens (solid curve). However, even for $N = 2$ the spatial intensity pattern is considerably distinct from the refractive case.

duced efficiency and the appearance of additional diffracted orders. These effects are illustrated in Fig. 2 for what could be called a conventional diffractive element or $p = 1$.

From the PSF results derived above, we now investigate the behavior of the diffractive lens as parameter p increases while Np remains fixed. This corresponds to a sequence similar to the one depicted in Fig. 1. In performing the calculations previously described we have assumed an error-free diffractive element. Although this is not entirely realistic, in the presence of small errors the results should still be valid. Some discussion of fabrication errors with respect to the diffractive–refractive transition can be found elsewhere.[4]

In Fig. 3 the evolution of the point-spread function as a function of the number of zones is presented. The behavior of the element can be analyzed from two important aspects: spatial distribution and spectral behavior. These characteristics of the diffraction pattern are certainly related, but as the number of zones decreases and p increases, they need not evolve in a similar fashion. Indeed, we note in Fig. 3 that as the diffractive lens tends to the refractive limit the diffractive point spread function tends to the corresponding refractive pattern. For instance, when the number of zones decreases from 32 to 16, the total energy on the first order is reduced while the second-order focal point undergoes an increase in efficiency followed by a dislocation toward the paraxial refractive focal point. Such a process continues as N decreases, indicating that the diffractive element tends to recover the refractive focal point for the wavelength in question. Such clustering indicates that, spectrally, the tuning of parameter p in fact leads to a more refractivelike diffractive lens, in accordance with the idea that a higher order improves polychromatic behavior.[5,6] In contrast, the spatial characteristics of the point-spread function are also relevant because they reflect the efficiency of each diffracted order and consequently provide important information on image contrast. As we can see in Fig. 3, even though increasing p tends to reduce the contribution of spurious orders, it seems that the diffracted lens

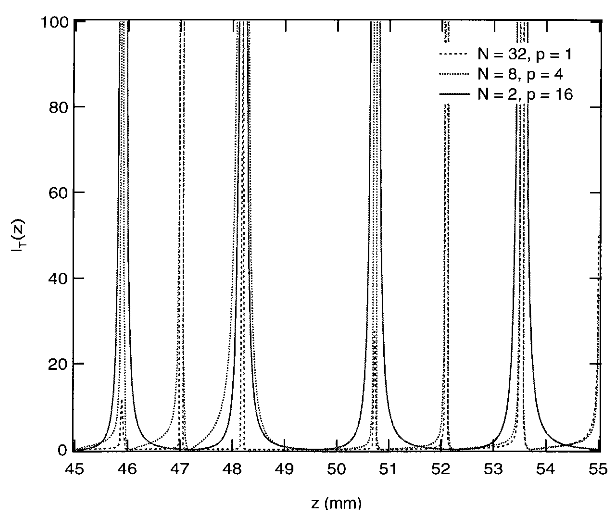

Fig. 4. Transition term $I_T(z)$, Eq. (15), as the diffractive element tends to the refractive limit. The same design parameters of Fig. 3 are adopted. In the purely refractive case, the transition coefficient is identically equal to one. As seen here, the basic dependence with the number of zones remains essentially unaltered, except for a scaling term, up to $N = 2$.

exhibits intensity profiles that are very peculiar up to the case in which only two zones are present. In this extreme limit, we might say that although parameter p tends to correct chromatic properties, it cannot correct the spatial distribution of the produced beam. As a result, from the spatial point of view the kinoform lens preserves its intrinsically diffractive nature with any nonunity number of zones. Such a result has also been verified numerically in other design cases.[4] This fact can be made more evident in Fig. 4, where a plot of $I_T(z)$, Eq. (15), is shown for distinct values of p. The basic dependence of the transition term, except for a scaling factor, is unaltered for all values of N.

3. Summary and Conclusions

The design of higher-order diffractive lenses poses the problem of the diffractive–refractive behavior in certain kinoform elements. We have expressed the on-axis scalar field of the diffractive lens in terms of an interference pattern of associated refractive lenses related to each individual zone and modulated by a zone-dependent complex-valued coefficient. Although for transverse fields the same idea should apply, it is not possible to obtain closed-form solutions as in the on-axis case. We have found that it is also possible to express the point-spread function of the diffractive lens as the product of a transition term and the intensity pattern of a refractive lens. In this form, the connection diffractive–refractive is immediately established.

Even though the refractive behavior becomes more evident as the maximum phase height increases, the diffractive lens preserves a unique spatial identity up to the point at which only two zones are present but tends continuously to the refractive spectral behavior. This result has also been observed for gratings by means of an altogether different method.[3] Although refractive elements without proper achromatization may be acceptable in some situations, for diffractive lenses there is greater difficulty because the amount of chromatic aberrations is dramatic. If the problem in question requires operation of a few discrete wavelengths, higher-order diffractive lenses provide an elegant solution. As the number of zones decreases and higher values of maximum phase are attained, the diffractive behavior approaches that of a refractive lens but never completely reaches it; in fact, for $N = 2$ the spatial intensity pattern of the lens departs even more significantly from the refractive behavior. Consequently, although intuitive, the concept of a transition between diffractive behavior and refractive behavior does not evolve identically in the spatial-spectral sense. In broadband applications, diffractive optics seem more promising for use as hybrid diffractive–refractive elements.

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